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Application of Markov Model and Standardized Precipitation Index for Analysis of Droughts in Bundelkhand Region of India

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ABSTRACT

Drought is a common occurrence in Bundelkhand, a drought prone region of Central India and agriculture is the primary economic sector affected. The present study is based on the 44 years rainfall data for the period 1968 to 2011 compiled from the meteorological observatory of CSWCRTI, Research Centre, Datia, Madhya Pradesh. In the present study we have used Standardized Precipitation Index (SPI) to assess the vulnerability of meteorological droughts in Bundelkhand region of India. The worst drought was observed during July, 1970 as indicated by the lowest ever SPI value of -2.48. Highest drought magnitude (5.93) has been observed during June to October 1979 with a return period of 96 years followed by May to August, 1987 with a magnitude of 3.43 and return period of 48 years. In the last decades maximum drought magnitude was observed during July–August, 1993 and in most recent past a drought magnitude of 1.59 was observed during July to October, 2009. The multi period Markov Model has been applied for prediction of occurrence of droughts in next five years in the region. Based on the drought analysis we conclude that there is a possibility of one severe drought of magnitude up to 2 within a period of 8 years. This information will be useful to agricultural planners and irrigation engineers for planning on long term drought mitigation strategy.

Keywords:

Bundelkhand, drought magnitude, multi-period markov model, standardized precipitation index,

INTRODUCTION

Drought is considered by many researchers to be the most complex but least understood of all natural hazards, affecting more people than any other hazard (Sivakumar et al. 2005). When thinking of natural hazards, droughts are often perceived by society to play a less dominant role compared to floods. Unlike the effects of a flood which can be immediately seen and felt, droughts build up rather slowly, creeping and steadily growing (Lehner et al. 2001). Drought is a three dimensional recurring natural phenomenon characterised by its severity, duration and areal extent, therefore difficult to assess. Researchers normally use drought index to assess the severity and magnitude of drought. Some of the widely used drought indices are the Palmer Drought Severity

Index (PDSI) (Palmer 1965), the Deciles (Kinninmonth et al. 2000), the Standardized Precipitation Index (SPI) (McKee et al. 1993) and the Reconnaissance Drought Index (RDI) (Tsakiris and Vangelis 2005). All these indices have their own capability to assess drought under different situations.

The Standardized Precipitation Index, known as SPI, is widely used as drought meteorological index to identify the duration and severity of drought (Angelidis et al. 2012). The SPI, seems to be the most popular among the existing simple indices for the estimation of drought because it is simple, spatially consistent in its interpretation, probabilistic so that it can be used in risk management and decision analysis, and can be tailored to time periods of user's interest (Edossa et al. 2010). The SPI is widely used for defining and monitoring meteorological droughts. It describes the behaviour of only one variable, precipitation, and for this reason, since its appearance, the SPI has become very popular due to its low data requirements. According to Tsakiris et al. (2007) three important characteristics of SPI are (i) it is uniquely related to probability, (ii) only precipitation is needed to calculate the precipitation deficit for the current period and for the desirable time scale and (iii) it is normally distributed so it can be used to monitor wet as well as dry periods.

This drought index is being extensively used among different workers throughout the world; among them some recent are Raziei et al. 2009, Vasiliades et al. 2011, Angelidis et al, 2012. Drought indices like SPI and others can assess the severity and magnitude of drought but they cannot forecast the future situation of drought. Different models like Markov model, Climgen etc. have the capability to do that. The Markov models are frequently proposed to quickly obtain forecasts of the weather states at some future time using information given by the current state. One of the applications of the Markov chain models is to forecast the occurrence of daily, weekly or monthly precipitation. In some recent works on Markov model, Srikanthan (2005) generated daily rainfall series using a nested transition probability model, Daniel (2008) used multiple-Markov model to forecast monthly rainfall and predicted drought for future months Selvaraj and Selvi (2010) applied stochastic process for describing and analysing the daily rainfall pattern at Aduthurai in Tamil Nadu.

The importance of these indices and models can be visualized if we apply them in an area where drought is a common phenomenon. The Bundelkhand Region of central India is such an area which is historically drought prone. It is one of the most backward regions in India and rates very low on almost all development indicators. It is predominantly an agrarian economy; over 80% of population is dependent on agriculture, livestock, usufructs from forest and outsourcing income by seasonal migration after Rabi sowing. Agriculture forms the backbone of the rural economy with almost 75% of the people dependent on agriculture and animal husbandry for their livelihood. Rainfall is the ultimate source of surface, ground, green and blue water resources for raising biomass and other utilities. Five out of seven districts had more than 50% rainfall deficit. All the districts experienced meteorological drought. Historically Bundelkhand region of UP and MP used to have one drought in 16 years in 18th and 19th centuries which increased by three times during the period 1968 to 1992 and the last decade witnessed continuous drought of four years (NRAA 2008).

It is not possible to avoid droughts, but preparedness can be developed and impacts can be managed. The success depends on how well the droughts are defined and drought characteristics are quantified. In this study, the severity and magnitude of drought was analyzed using SPI and a futuristic precipitation scenario has been developed using Markov model in a representative area of Bundelkhand region.

MATERIALS AND METHODS

Dataset

The present study is based on daily rainfall data recorded at the meteorological observatory of Central Soil and Water Conservation Research and Training Institute, Research Centre, Datia, Madhya Pradesh. The observatory is situated at 25°42'12.25" N latitude and 78°25'58.87" E longitude with 222 m above MSL. The daily rainfall data for the last 44 years (1968-2011) were

Table1: Drought classification by SPI value and corresponding event probability (Lloyd-Hughes and Saunders 2002)

SPI value	Category	Probability (%)
SPI ≥ +2.0	Extremely wet	2.3
1.50 ≤ SPI ≤ 1.99	Severely wet	4.4
1.00 ≤ SPI ≤1.49	Moderately wet	9.2
0.00 ≤SPI ≤0.99	Mildly wet	34.1
−0.99 ≤SPI ≤0	Mild - drought	34.1
−1.49 ≤ SPI ≤ −1.00	Moderate drought	9.2
−1.99 ≤ SPI ≤−1.5	Severe drought	4.4
SPI ≤−2.00	Extreme drought	2.3

Table 2: Drought periods with the higher drought magnitudes of Datia

Year	Month	Drought Magnitude	Rank	Return Period (Years)
1979	Jun - Oct	5.83	1	96.00
1987	May - Aug	3.43	2	48.00
1993	Jul - Aug	3.29	3	32.00
2007	Jul - Oct	2.99	4	24.00
1970	Jul - Aug	2.86	5	19.20
1972	May - July	2.75	6	16.00
1986	Aug - Sep	2.46	7	13.71
2016	Aug - Sep	2.37	8	12.00
1980	Sep	1.96	9	10.67
1991	Sep - Oct	1.93	13	9.60
2002	Jun - Jul	1.90	14	8.73
1981	Jun - Jul	1.88	15	8.00
1968	Aug - Sep	1.84	16	7.38
1990	Jun - Jul	1.73	17	6.86
2012	Jun - Jul	1.61	18	6.40
200 9	Jul - Oct	1.59	19	6.00
2005	Aug - Oct	1.58	20	5.65

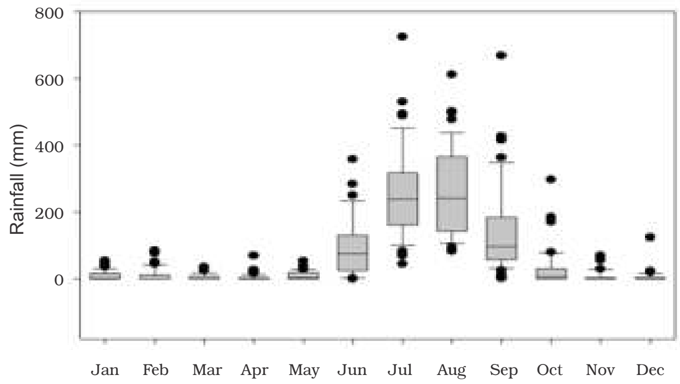

Fig. 1. Distribution of monthly rainfall in Bundelkhand region

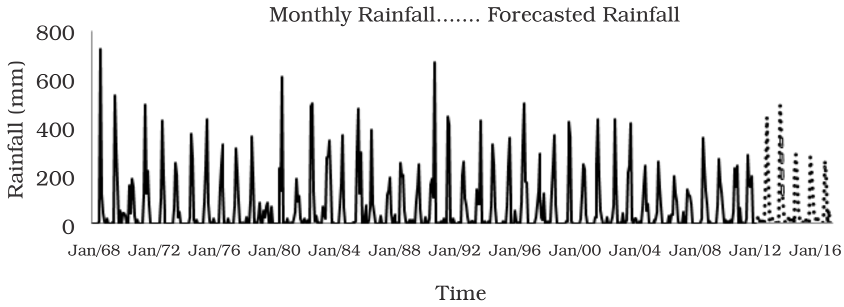

Fig. 2. Time series plot of observed (1968-2011) and forecasted (2012-2016) monthly rainfall

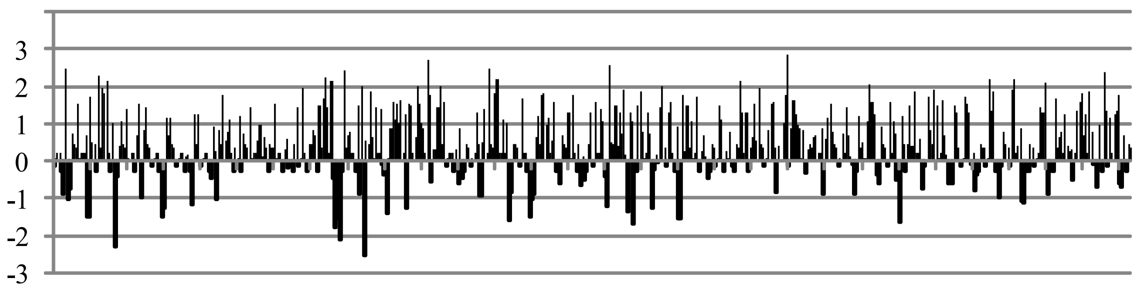

Fig. 3. Distribution of monthly SPI values of Bundelkhand region

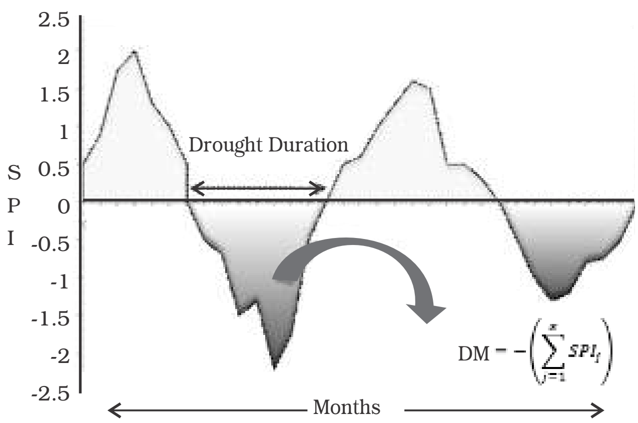

Fig. 4. Drought Duration and Drought Magnitude (DM) based on the SPI index

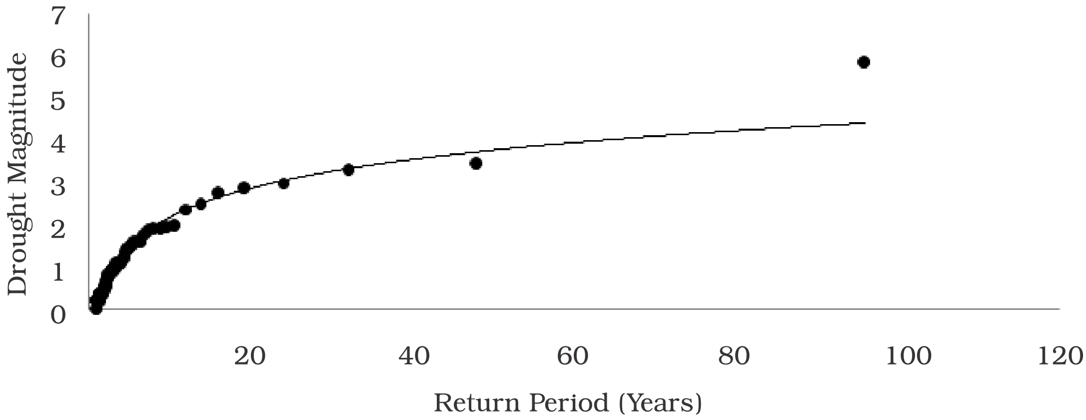

Fig. 5. Relationship between drought magnitude and corresponding return period

compiled and used to study and the distribution of monthly rainfall (Fig. 1).

Multi-period Markov model

Multi-period Markov model was used to reflect different seasonal or monthly means, which requires double indexing (year, month) subscripts (Haan, 1982). This is based on the assumption that rainfall is essentially a random variable. Hence it is possible to predict rainfall data by statistical methods. To predict the rainfall data for the next five years viz. 2012-2016, the following formula was used:

$$Q_{i,j} = \bar{Q}_j - b_j\left(Q_{i-1,j-1} - \bar{Q}_{j-1}\right) + t_i\sigma_j\left(1 - r_j^2\right)^{1/2}$$

where,

I is the periodic index (year(s)), i = 1968, 1969, ..., 2011

j is the annual index (month(s), j = 1, 2, ..., 12

$Q_{i,j}$ is the predicted precipitation value (which will be set to zero, in case it is negative, but only after all the subsequent rainfall values have been generated)

Q_j is the mean precipitation in j-th month

b_j is a parameter given by $r_j\left(\frac{\sigma_{j+1}}{\sigma_j}\right)$

$Q_{i-1,j-1}$ is the final recorded monthly precipitation from the preceding year or the generated value from the preceding year.

Q_{1-j} is the preceding mean monthly rainfall

t_i is a random number selected from a normal distribution having a zero mean and unit variance

σ_j is the standard deviation of observed precipitation values for the j-th month$j$

r_j is the correlation coefficient between the values in the period $j+1$ and period j

The parameters illustrated above were fed into equation (1) to come up with the generated precipitation values.

The Standardized Precipitation Index (SPI)

The impact of rainfall deficiency on water resources varies markedly on a temporal scale for different water storage components of the hydrologic system. While soil moisture responds to precipitation anomalies on a relatively short scale; groundwater, stream-flow, and reservoir storage reflect longer-term precipitation anomalies. McKee et al. (1993) developed the SPI to quantify the precipitation deficit for multiple time scales, reflecting the impact of precipitation deficiency on the availability of various water supplies. The SPI provides a quick and handy approach to drought analysis. Other advantages of this approach are its relative simplicity and minimal data requirements.

We applied the procedure given by Mckee et al. (1993) for computation of SPI. The outline of the procedure used is given below.

Computation of the SPI involves fitting a gamma probability density function to a given time series of precipitation data, whose probability density function is given by the expression:

$$g(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} \qquad (2)$$

Where a >0 is a shape parameter, b >0 is a scale parameter, and *x* >0 is the amount of precipitation. G(a) is the gamma function, which is defined as:

$$\Gamma(\alpha) = \int_0^{\infty} y^{\alpha-1} e^{-y} dy \qquad (3)$$

Fitting the distribution to the data requires a and b to be estimated. Using the approximation of Thom (1958), these parameters can be estimated as follows:

$$a = \frac{1}{4A}\left(1 + \sqrt{1 + \frac{4A}{3}}\right), \beta = \frac{\bar{x}}{\alpha}, \quad \text{with } A = \ln(\bar{x}) - \frac{\sum \ln(x)}{n}$$

where *n* represents the number of observations. Integrating the probability density function with respect to *x* results the following expression G(*x*) for the cumulative probability:

$$G(x) = \int_0^x g(x)dx = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} \int_0^x x^{\alpha-1} e^{-x/\beta} dx \qquad (5)$$

By putting the value of t=*x*/b in equation (5) we get

$$G(x) = \frac{1}{\Gamma(\alpha)} \int_0^x t^{\alpha-1} e^{-1} dt \qquad (6)$$

It is possible to have several zero values in a sample set. In order to account for zero value probability, since the gamma distribution is undefined for *x*=0, the cumulative probability function for gamma distribution is modified as:

$$H(x) = q + (1-q)G(x) \qquad (7)$$

where *q* is the probability of zero precipitation.

Finally, the cumulative probability distribution is transformed into the standard normal distribution to yield the SPI. Following the approximate conversion provided by Abramowitz and Stegun (1965), it results:

$$z = SPI = -\left(t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3}\right), t = \sqrt{ln\left(\frac{1}{(H(x))^2}\right)}$$

for 0 < H(x < 0.5 (8)

$$z = SPI = +\left(t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3}\right), t = \sqrt{ln\left(\frac{1}{(1.0 - H(x))^2}\right)}$$

for 0.5 <H(x) < 1.0 (9)

and c_0 = 2.515517; c_1 = 00802853; c_2 = 0.010328d_1 =1.432788; d_2 = 0.189269; d_3 = 0.001308

Once standardized, the strength of the SPI, as given in the following Table 1 (Lloyd-Hughes and Saunders 2002) can be visualized. Positive SPI values indicate greater than median precipitation, and negative values indicate less than median precipitation. Since the SPI is normalized, wetter and drier climates can be represented in the same way, and wet periods can also be monitored using the SPI.

Drought Magnitude

Different time scales reflect the impact of drought on the availability of the different water resources. To calculate the drought magnitude, the procedure given by Mckee et al. (1993) has been adopted. Drought event occurs at any time when the SPI is continuously negative and reaches an intensity of -1.00 or less and the event ends when SPI becomes positive. In this method the positive SPI values were set to zero and negative SPI values were multiplied by negative one. The drought magnitudes were calculated as the positive sums of the SPI values that are always negative over consecutive months i.e.

$$\text{Drought Magnitude} = -\left(\sum_{j=1}^{x} SPI_j\right) \qquad (10)$$

where SPI_j is the negative SPI values running continuously over *x* months.

RESULTS AND DISCUSSION

Futuristic forecast of rainfall using Markov model

Rainfall data for the period 2012-2016 were predicted using multiple Markov chain model. Datia rainfall station had 44 years of record ranging from 1968 to 2011. The Markov generation technique was used to generate data from 2012 to 2016 so that future simulations could be made. In using the Markov generation technique, the following procedure was followed.

In calculating the parameters of equation (1), it was assumed that there was no data after 2011. Monthly rainfall values of 2011 were used as initial values in equation (1) to generate values for the next year (2012). The actual monthly rainfall from 1968 to 2011 along with the forecasted rainfall for 2012-2016 has been shown in Fig. 2. The forecasted result indicated that there can be drought like situation during the next five years.

Drought analysis

A meteorological drought is defined in terms of monthly rainfall shortfall over long term monthly mean. The monthly rainfall pattern over time indicates that the study area had a net monthly moisture deficit. This will be a first impression of how urgently this area needs reliable water sources. To characterize drought, SPI was used and the same was calculated by following the method given in Materials and Methods section. Plot of the SPI values have been shown in Fig. 3. Minimum SPI was found in July, 1970 with SPI = -2.48 followed by -2.05 which has been observed for the forecasted period during September, 2016. During the last two decades maximum draught in terms of severity has been observed with SPI =-1.74 in July, 2002.

If the SPI values continuously remain negative for consecutive months, the effects of drought become more harmful. To measure this effect, drought magnitude was calculated. The draught magnitude and drought duration obtained for Datia has been shown in Fig. 4. During the last 44 years, highest drought magnitude (5.93) has been observed during June to October, 1979 with a return period of 96 years followed by May-August, 1987 with a magnitude of 3.43 and return period of 48 years. In the last two decades maximum drought magnitude was obtained in July-August, 1993 and in most recent past a drought magnitude of 2.99 and 1.59 was observed in July-October, 2007 and 2009 respectively. These findings are in conformity with those of Singh, 2011 (http://www.bundelkhand.in). For the next five years, droughts in 2016 and 2012 have been forecasted with drought magnitude of 2.37 and 1.61 respectively. Drought periods with the higher drought magnitudes (Drought magnitude >1.5 i.e. Extreme drought) have been shown in Table 2 along with their return period.

Plots for drought magnitudes against return period were made and are as shown in Fig. 5. The figure illustrates that, drought magnitudes and the return periods associated with them are related such that large drought magnitudes are associated with large return period. Fig. 4 clearly indicated that more than 80 per cent of the droughts having magnitude of less than 2 with the return periods of less than 8 years (NRAA, 2008). Therefore people can expect one drought of magnitude up to 2 within a period of 8 years.

Drought monitoring and forecasting are essential tools for implementing appropriate mitigation measures in order to reduce negative impacts. In this Study, the SPI was proved to be a useful index for finding out drought severity and magnitude in Bundelkhand region. The droughts in Bundelkhand over the study period vary from moderately high to extremely high according to the calculated SPI values. Drought magnitudes for Bundelkhand spread from one month (with magnitude 1) to five months (with magnitude 5.83). This can cause severe effects if it occurred without planning for the drought. The region may expect one drought of magnitude up to 2 within a period of 8 years. There is a huge scope to suggest appropriate drought mitigation measures and constraints to proper drought management in the region. There is more wisdom in planning, forecasting and preparing for a drought than do nothing and suffer the consequences.

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